Stocks & Shocks

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ESD 862 Modeling Risk, Dynamics & Decisions

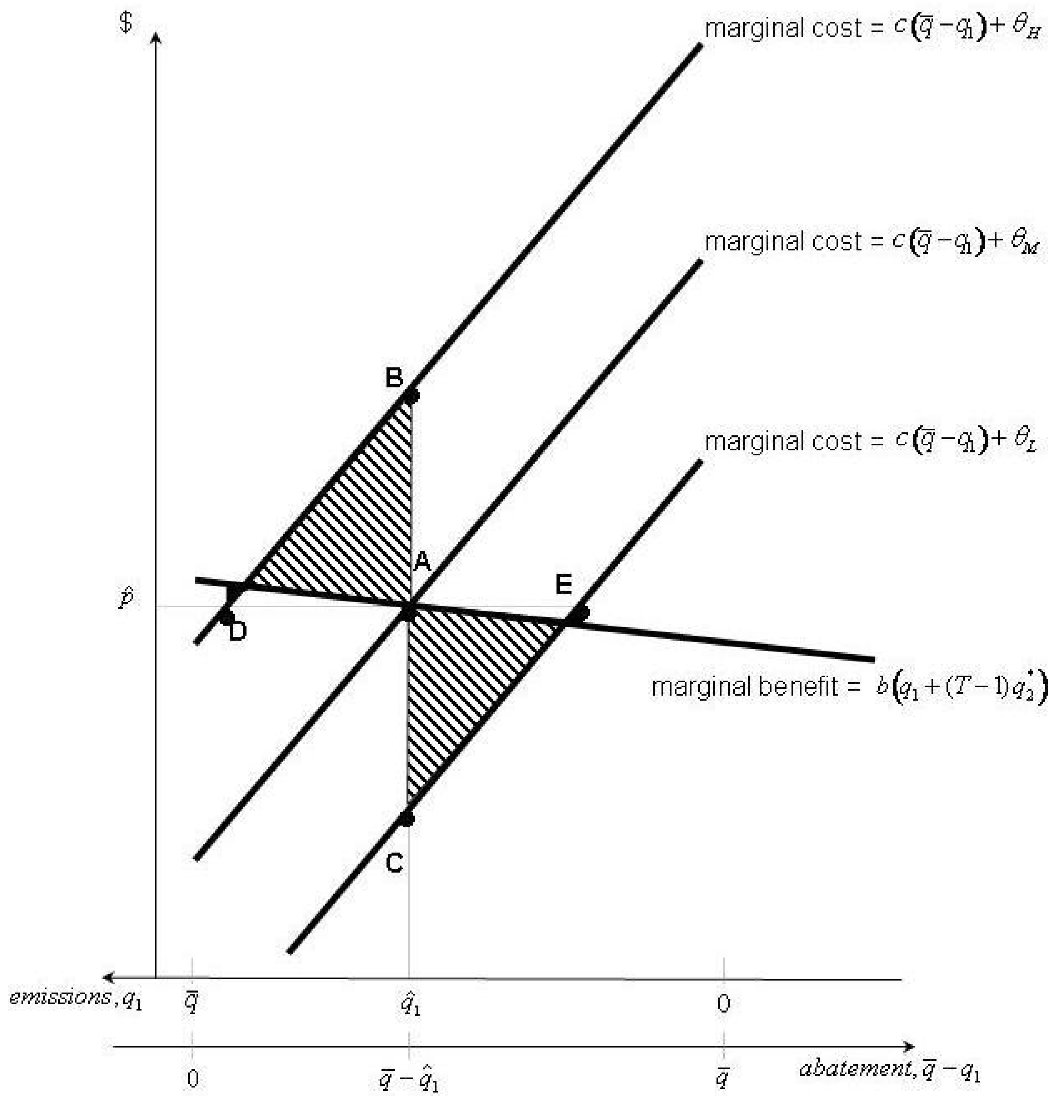
Outline

- Weitzman's Problem
  - Regulation and private information
  - Application to the greenhouse gas problem: stock pollutant
- Dynamic Version
  - The Dynamics of Uncertainty
  - Discrete Time Examples
  - Continuous Time Examples
- Mapping the Model to the Public Policy Debate

Prices v. Quantities in the Context of the Greenhouse Gas Problem

- Original Weitzman result:
  - Regulate using prices if marginal cost curve is steeper than marginal benefit curve.
  - Regulate using quantities if marginal benefit curve is steeper than marginal cost curve.

Marginal Costs and Marginal Benefits of Abatement ala Weitzman (1974)



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- What’s special about the Greenhouse Gas problem?
  - Stock pollutant.
  - Marginal benefit curve is flat, marginal cost is steeper.

Analysis of Carbon as a Stock Pollutant ala Pizer (2002)

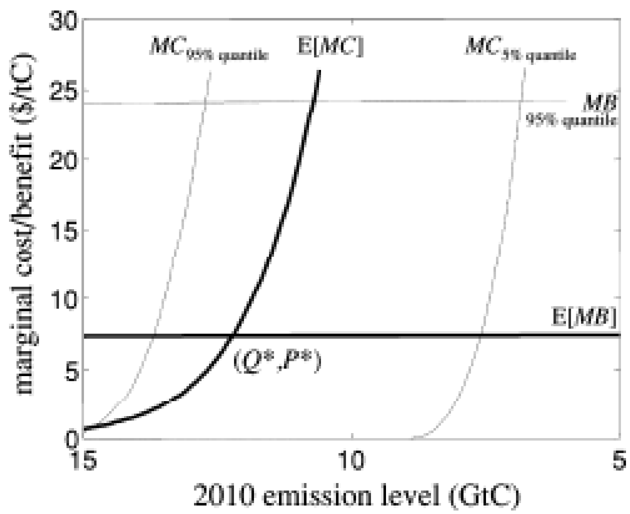

Fig. 2. Distribution of marginal costs and benefits in 2010. (The 5% quantile of marginal benefits overlaps the *x*-axis.)

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- What's special about the Greenhouse Gas problem?
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- Commonly drawn conclusion: regulate GHGs using prices not quantities, i.e., carbon tax and not cap & trade.

My Argument / Clarification

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  - A realization of a shock in one year shifts the conditional forecasts in all future years,
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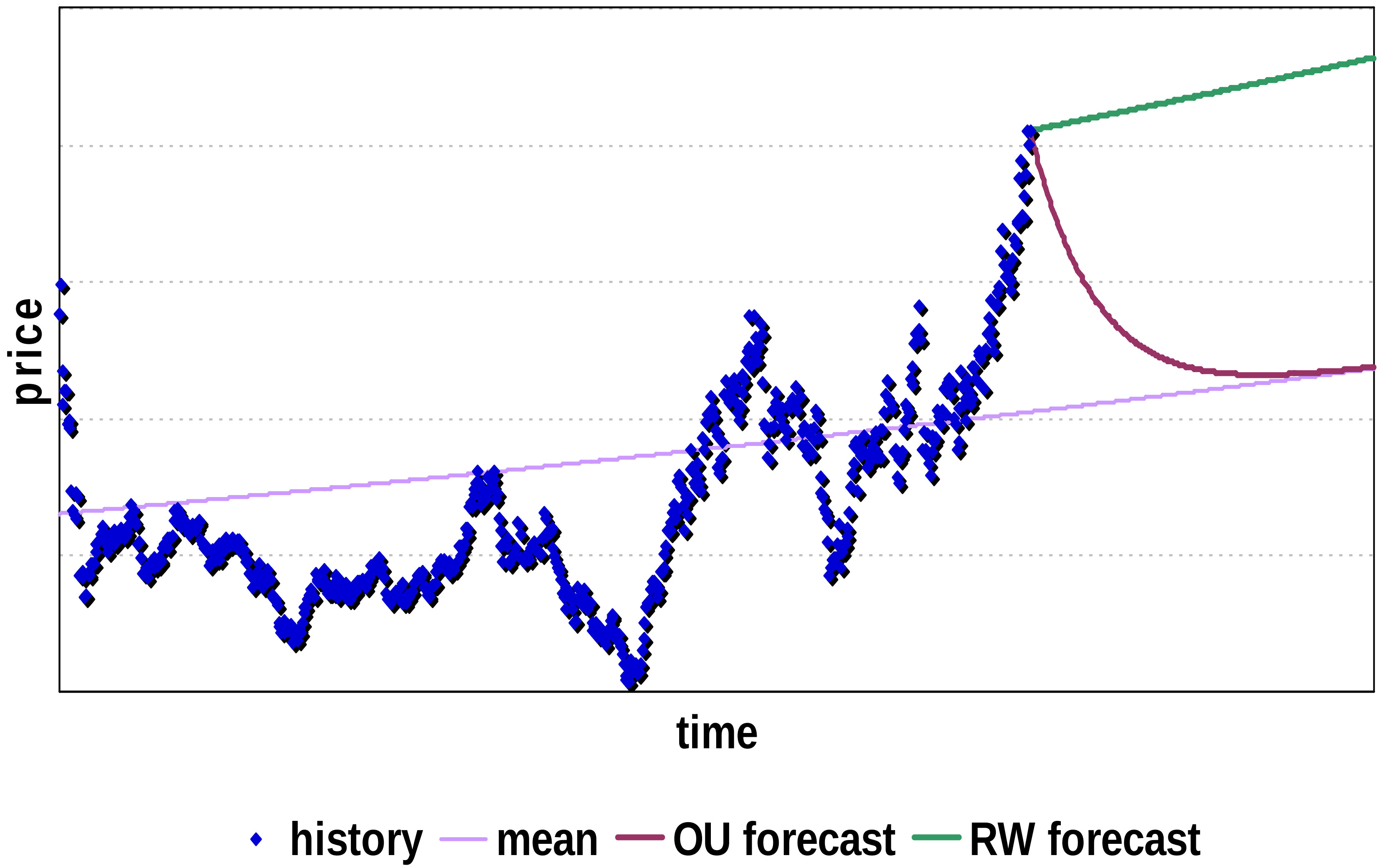
  - Cannot examine the marginal benefit function for this year's emissions locally to the current level.
- The "stock pollutant" feature of CO_2 is not dispositive in the P v. Q debate.

Dynamics of Uncertainty

Dynamics of Uncertainty

- Temporary Shocks
  - Mean reverting process, such as Ohrenstein-Uhlenbeck or White Noise
- Permanent Shocks
  - Random walk process, such as Geometric Brownian Motion
- The Role of Conditional Forecasts
  - What does today’s news about costs say about tomorrow’s likely cost?

Differing Conditional Forecasts



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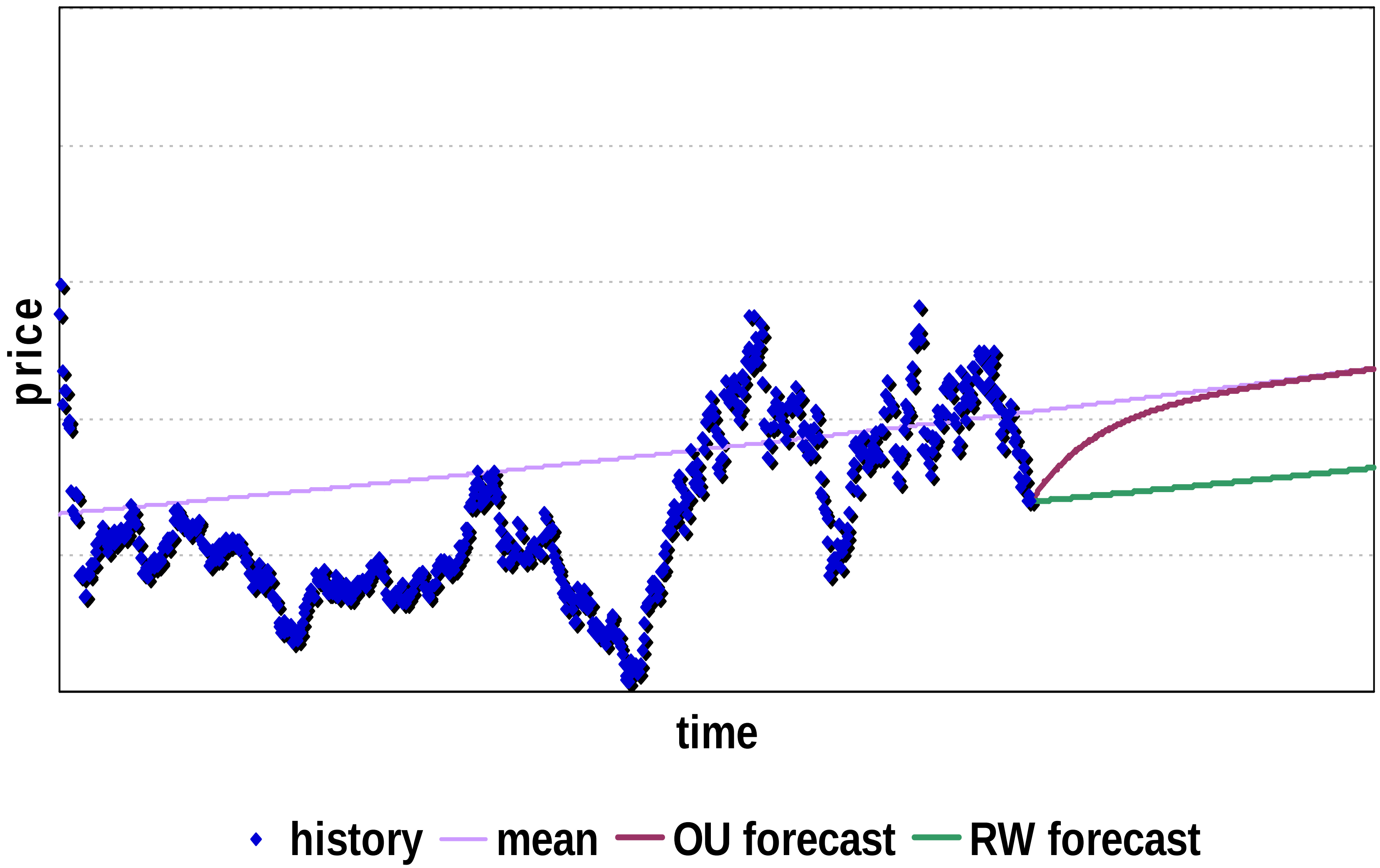


Discrete Time Examples

Example Set-up: Benefits & Costs

- T periods, no discounting
  - q_t -- per period emissions
  - Q_t -- aggregate emissions to date

- $B(Q_T) = -(b/2)Q_T^2$, $b>0$
  - benefits are avoided damages
  - damages due to aggregate emissions, modeled as a "settling up" at the end of time, T

- $C(q_t,\theta_t) = (c/2)(q_t-\bar{q})^2 - \theta_t(q_t-\bar{q})$, $c>0$
  - $\bar{q}$ is a reference level of emissions
  - costs are minimized at $q_t=\bar{q}+(\theta_t/c)$
  - θ_t is the uncertain parameter

Modeling Dynamics

- Within Each Period
  - first, the regulator sets rules, e.g., fix $\hat{p}_t$ or $\hat{q}_t$
  - next, the period's value for θ_t is realized as private information to the firm's
  - finally, firm's choose production level, q_t

- Period to Period
  - at the conclusion of each period, the value of θ_t becomes common knowledge
  - in the next period, the regulator can reset the rules conditional on events in the previous period

Uncertainty Dynamics

- Simplify the problem…focus all uncertainty in time period 1
- Case A -- Pure Temporary Shock:
  - θ_1 is uncertain, but
  - at start of t=1, $\theta_2,\ldots\theta_T$ are known/fixed, i.e., independent of the realization of θ_1
  - analogous to a mean reverting process in re conditional forecasts
- Case B -- Pure Permanent Shock:
  - θ_1 is uncertain, and
  - at start of t=1, $\theta_2,\ldots\theta_T$ are unknown, but
  - at end of t=1, $\theta_2,\ldots\theta_T$ are known, $\theta_2 = \theta_3 = \ldots\theta_T = \theta_1$
  - analogous to a random walk process in re conditional forecasts
- Elimination of uncertainty at t=2 is a useful simplification that preserves the key issues

Backward Programming

- With the elimination of uncertainty at the conclusion of period 1, it is straightforward to solve the optimal sequence of outputs for periods 2,…T and to enforce them
  - moreover, $q_2^* = q_3^* = \ldots = q_T^*$

- In Case A, the optimal sequence of outputs will depend upon q_1, and will be independent of θ_1,

$$q_2^*(q_1)$$

- In Case B, the optimal sequence of outputs will depend upon q_1, and will depend on θ_1 via $\theta_2, \ldots \theta_T$,

$$q_2^*(q_1, \theta_1)$$

Correct First Period Optimization

- Define the value function, $V(q_1,\theta_1)$:

$$V(q_1,\theta_1) = B(q_1+(T-1)q_2^*(\cdot)) - (T-1)\ C(q_2^*(\cdot),\theta_2(\cdot))$$

- Optimal output in period 1:

$$\max\ [-C(q_1,\theta_1) + V(q_1,\theta_1)]$$

- First order condition:

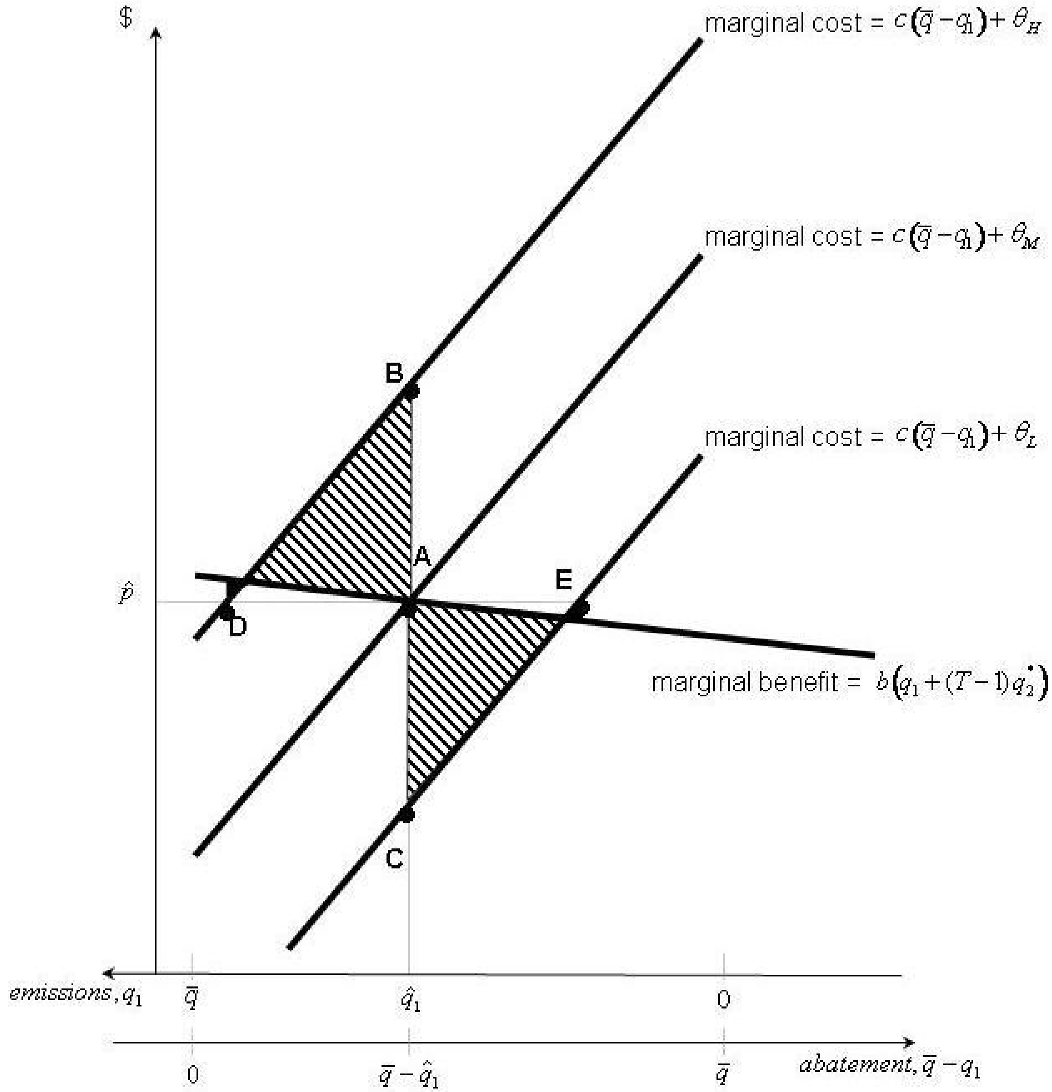
$$- \partial C(q_1,\theta_1)/\partial q_1 = \partial V(q_1,\theta_1)/\partial q_1$$

- $\partial V(q_1,\theta_1)/\partial q_1 = b(q_1+(T-1)q_2^*(\cdot))$
  - case A: $= b(q_1+(T-1)q_2^*(q_1))$
  - case B: $= b(q_1+(T-1)q_2^*(q_1,\theta_1))$

First Period Optimization: Traditional Comparison

- Maximize $[-C(q_1,\theta_1) + B(Q_T)]$
- First order condition: $\partial C(q_1,\theta_1)/\partial q_1 = dB/dQ$
- Examine variations in θ_1 and therefore the optimal choice of q_1^*
- Either the first period is the whole problem…T=1, or ignore dependence of q_2^*, q_3^*,… q_T^* on q_1 and on θ_1

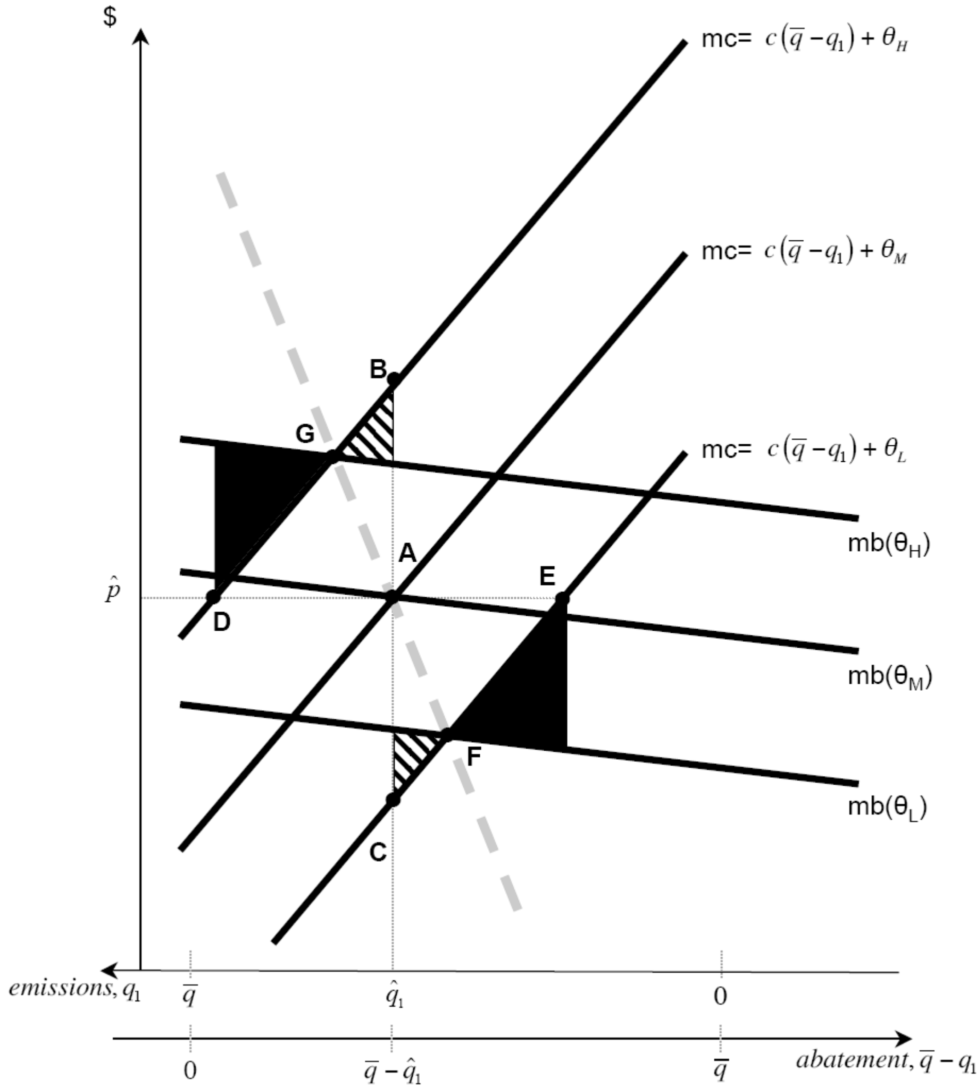
Marginal Costs and Marginal Benefits of Abatement ala Weitzman (1974)



The Central Problem

- Depending upon the information structure, variations in θ_1 may also change the optimal outputs in q_2^*, q_3^*,… q_T^*.
- In evaluating how variations in θ_1 should impact variations in q_1^*, it is not necessarily correct to evaluate the marginal benefit function locally around the original level of aggregate output.
- The traditional argument for using price controls for a stock pollutant like carbon ignores the fact that surprisingly high current costs may also foreshadow high future costs.

Figure 3: Marginal costs and marginal benefits for Case B



Continuous Time Examples

Example Set-up

- $C(q_t,\theta_t) = \theta_t \exp(-q_t)$,
  - θ_t is the uncertain parameter
  - $C(0,\theta_t)=\theta_t$, i.e., there is a maximum cost
  - $\{q_t \to \infty\} \Rightarrow \{C \to 0\}$
  - no restrictions on q_t, i.e., q_t,<0 is possible, but at great cost, and Q_t>Q is possible for t<T.

- Key Finesse: Don't evaluate benefits…only look at the problem of minimizing the cost of achieving a given bound on emissions, Q

Solution to the Certainty Case

- Start with the model under certainty, where θ_t grows at rate ν and assuming a discount rate, r.

- The cost minimizing emissions path has
  - …emissions growing linearly in time, so that the marginal cost grows exponentially at the discount rate, r
  - …the linear growth rate is ν-r.

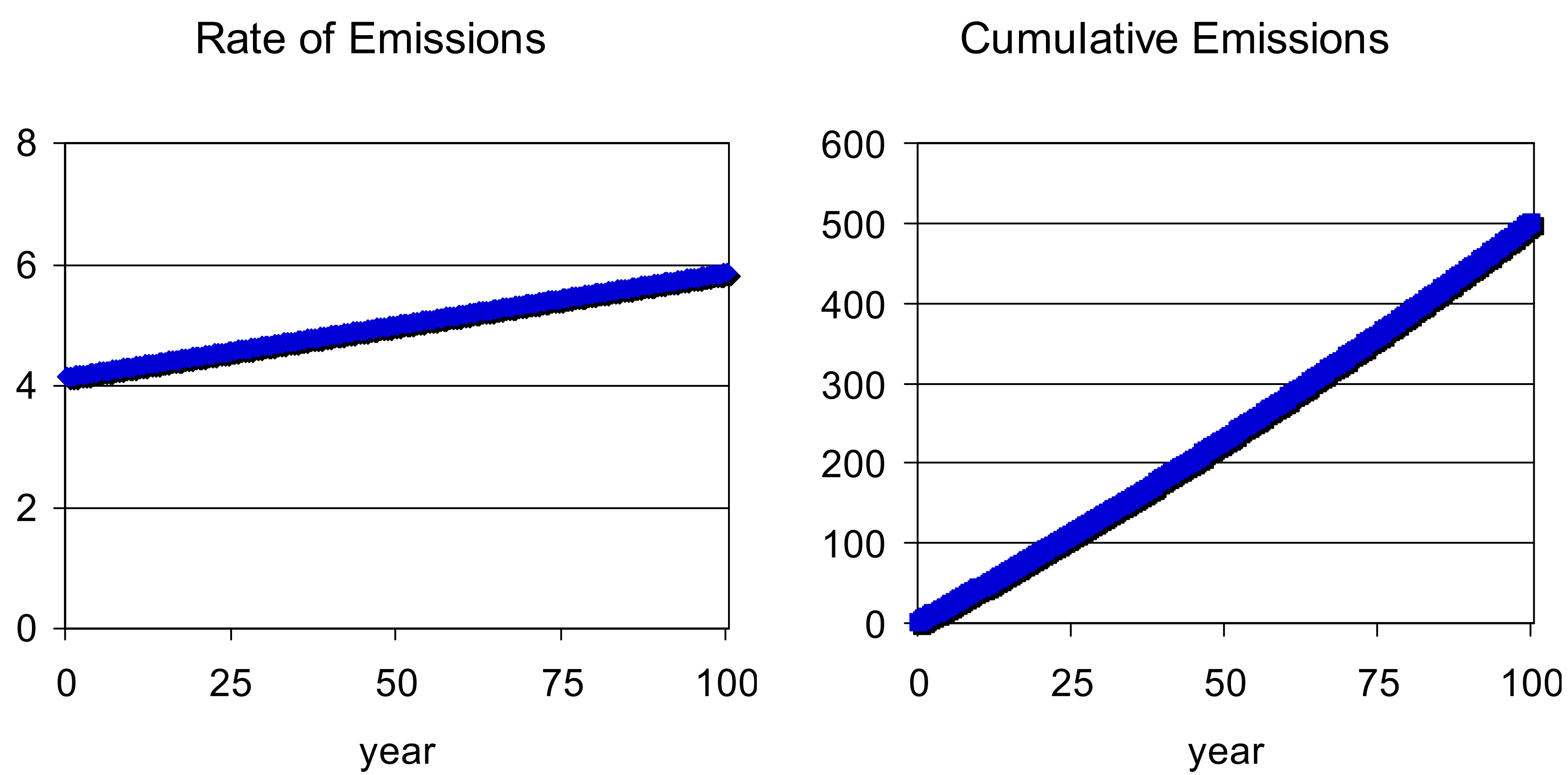
- The initial level of emissions is set so that allowed emissions are exhausted at the last instant in time

$$q_0 = Q_0/T - 0.5*(\nu-r)T$$

- Therefore, the optimal emissions path in the certainty case is

$$q_t = q_0 + (\nu-r)\,t$$
$$= Q_0/T - 0.5*(\nu-r)T + (\nu-r)\,t$$

Cost Minimizing Emissions Path in the Certainty Case



What If?
Analyze off equilibrium paths in the certainty case

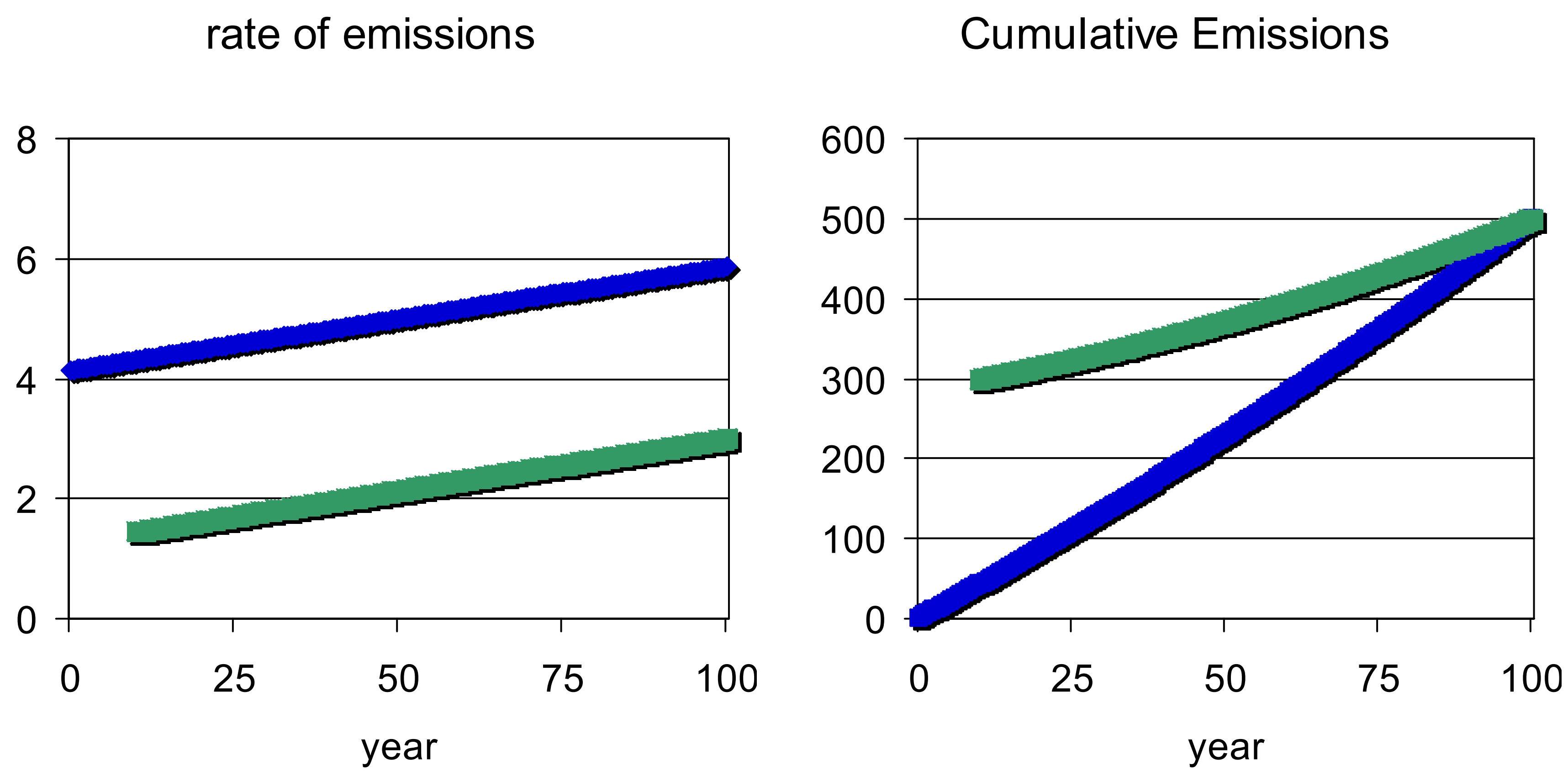
- Suppose we are at time t, and emissions to date have not followed the cost minimizing path. What is the cost minimizing path for the remaining time given the current level of aggregate emissions?
- A straightforward generalization…
- Today's cost minimizing emissions are…

$$q_t = K(Q_t,T-t) = Q_t/(T-t) - 0.5*(\nu-r)(T-t)$$

  - i.e., as if looking forward, $\tau>t$, we hoped to follow the path…

$$q_\tau = Q_t/(T-t) - 0.5*(\nu-r)(T-t) + (\nu-r)\,\tau$$

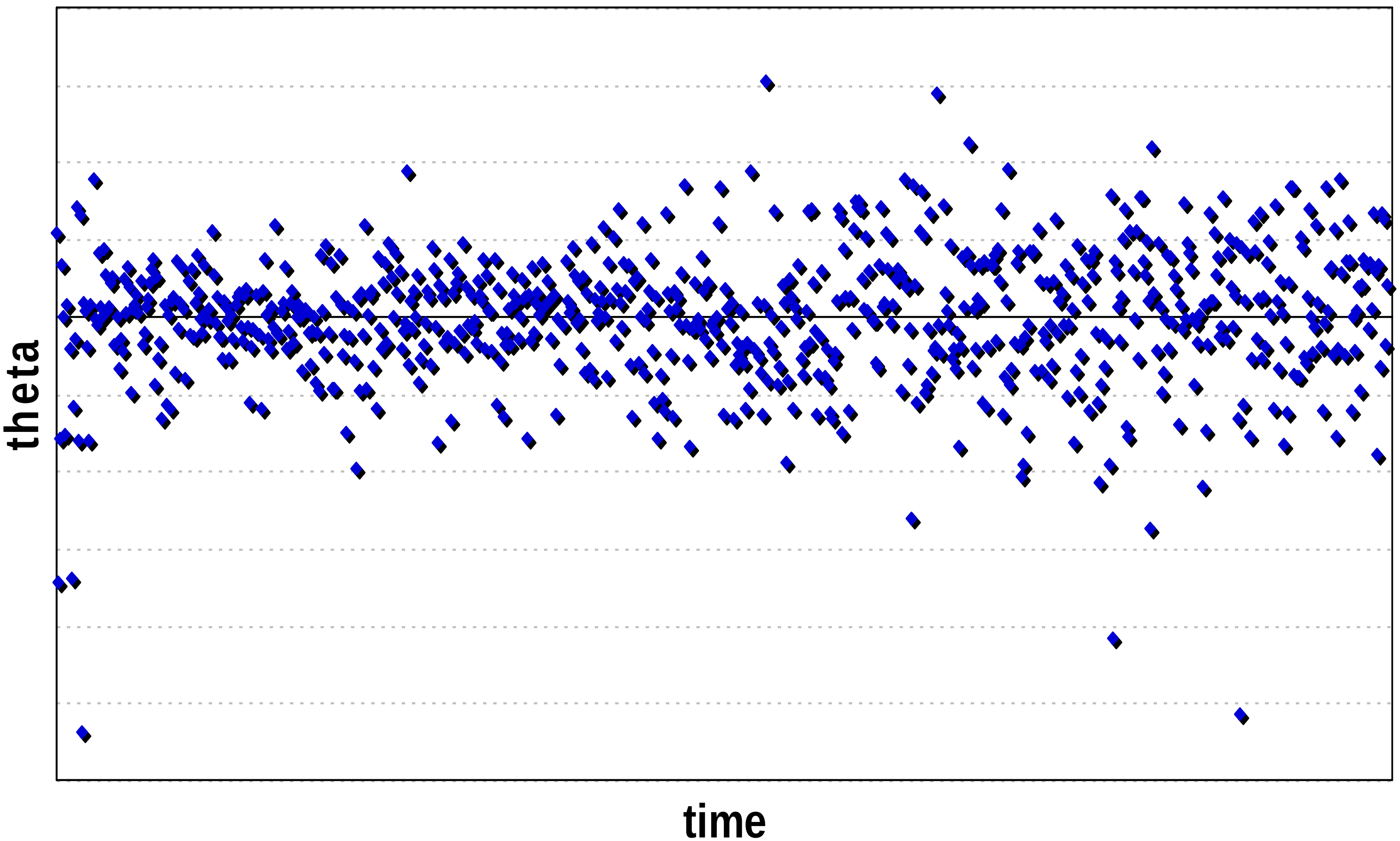
“What If” Cost Minimizing Emissions Path Following Period of Excess Emissions



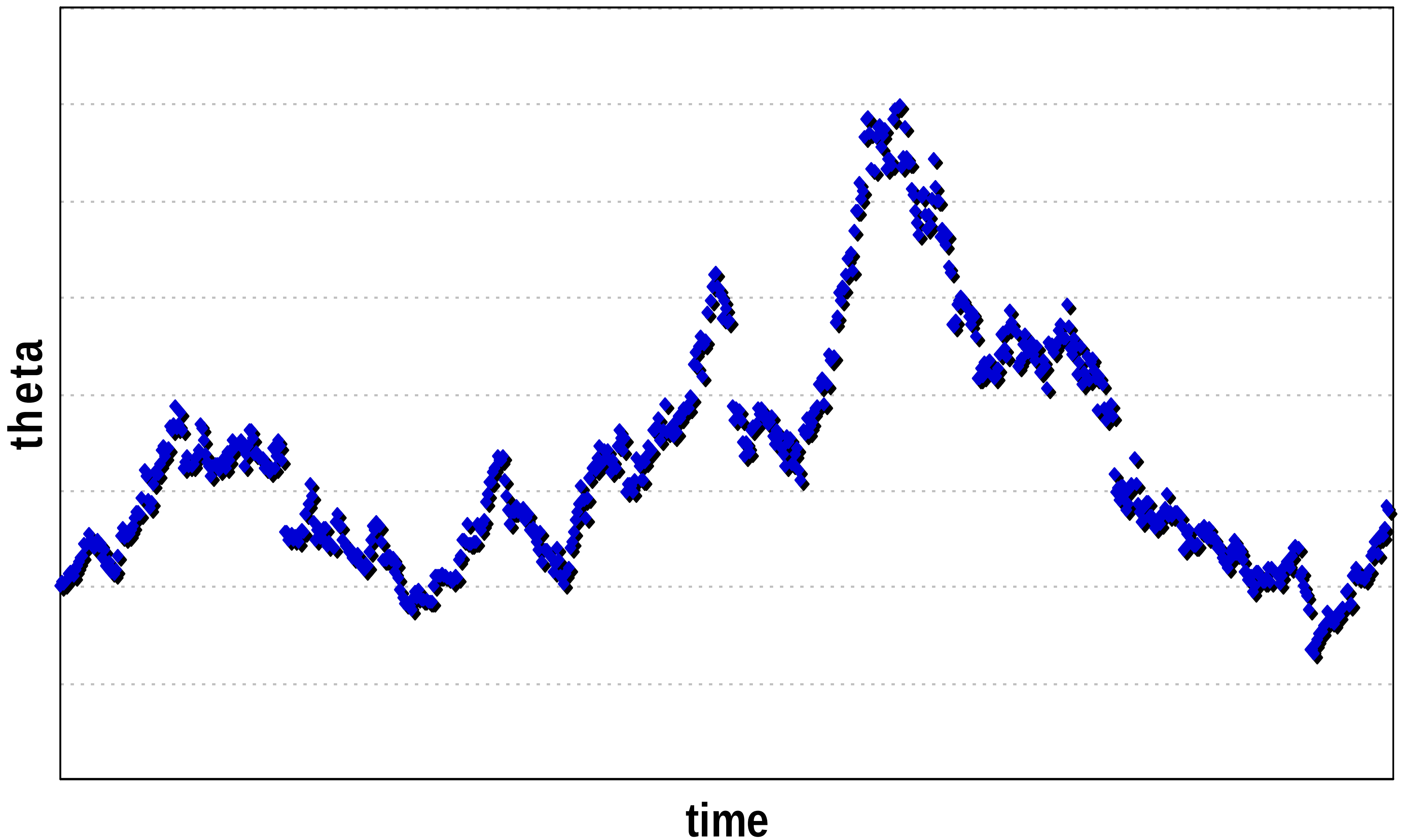
Uncertainty Dynamics

- Case C -- Pure Temporary Shock
  - Uncertainty is white noise: $\theta_t = \theta_0 \exp(\nu t) + \sigma\, dz$
- Case D -- Pure Permanent Shock
  - Uncertainty is geometric Brownian motion: $d\theta_t/\theta_t = \mu\, dt + \sigma\, dz$

White Noise: Sample Path Observed Discretely



Geometric Brownian Motion: Sample Path Observed Discretely



Solution to Case C, Temporary Uncertainty

- In the face of a shock to the cost function, $dz \neq 0$, it is optimal to allow this period's emissions to adjust to the shock, completely

$$q_t^* = K(Q_t, T-t) + \sigma\, dz$$

- Variation in today's emissions, $\sigma\, dz$, is made up for by adjusting the target emission level in all subsequent periods, τ. This occurs through the adjustment of future target emissions based on aggregate emissions up to date τ, $K(Q_\tau, T-\tau)$.

- Aggregate emissions follow the process

$$dQ_t = K(Q_t, T-t) + \sigma\, dz$$

A mean reverting process, where the strength of reversion increases with the realized tightness of the cap and the shortness of time to the horizon.

Prices v. Quantities in Case C

- Instantaneously, all of the uncertainty is reflected in variation in the quantity of emissions;
- Instantaneously, there is no uncertainty about the shadow price of the optimal level of emissions;
- Price controls can always be used to implement the cost minimizing path. At each instant a price can be fixed based on cumulative emissions to date, without regard to the realization of the cost parameter, and the quantity can be allowed to be set optimally against this price given knowledge about the realized cost parameter:

$$P_t = E[q_t] \exp(-K_t(Q_t)) \neq q_t \exp(-q_t)$$

- Note, however, that the price level must be regularly updated, and indeed must be used to help make up for earlier "excess" emissions.
- Strict quantity controls can never implement the cost minimizing path since output in each instant of time needs to be responsive to the current realization of the shock.

Solution to Case D, Permanent Uncertainty

- The emissions path remains as in the certainty case,

$$q_t^* = K(Q_t, T\text{-}t)$$

- At any given instant, in the face of a shock to the cost function, $dz \neq 0$, it is *NOT* optimal to allow this period's emissions to adjust to the shock *at all*.
- Aggregate emissions follow a fixed, deterministic path, independent of the sequence of shocks
- Marginal cost varies, instant by instant, reflecting the evolution of the uncertain cost parameter, θ_t. Marginal cost follows a geometric Brownian motion.

Prices v. Quantities in Case D

- Price controls do not implement the cost minimizing path.
- Quantity controls, specified period-by-period, can be used to implement the cost minimizing path.
- Note this does not speak to the optimal path, since we have not addressed the weighing of costs and benefits.

Mapping the Prices v. Quantities Debate onto the Tax v. Cap Policy Space

Dynamic Models & the Rate of Information Flow

- The issue at hand is an assumed regulatory delay… controls are specified, uncertain variables are realized, private actors observe them while public actors don't, and actions are taken based on controls specified ex ante.
- Obviously, in the context of a dynamic model, with repeat performance, the question arises, how long is the regulatory delay? How long before the regulator eventually observes the cost parameter and can re-adjust the control parameter?
- This question is especially relevant when debating the stock pollutant argument, since the time frame is many decades long. Regulations will be adjusted. Much interim cost information will be observed along the way.
- The existing models have never raised this question, at all.

Cap & Trade & Banking and Borrowing

- As debated in the theoretical economics literature, quantity controls are always period-by-period controls.
- The discussion focuses on annual time frames and poses a fixed annual cap on carbon emissions against a fixed annual tax on carbon.
- But actual carbon cap&trade proposals allow banking of allowances across years.
- Cap & trade with frictionless banking and borrowing of allowances through time implements the cost minimizing emissions path in *BOTH* case C and D type uncertainty.
- The economists' comparison of PvQ controls and Cap v. Tax policy sets up a straw man debate that is not relevant to the actual policy choices.
- The real question is a lower order issue of how frictionless across years are emissions markets, and what institutional features need to be redesigned to improve the effectiveness of banking and borrowing.

Cap & Trade & Banking and Borrowing (cont.)

- Tax advocates assert that existing emissions market prices exhibit “too much” volatility. A tax can be readily fixed to a constant number.
- This assumes all uncertainty is like Case C.
- In Case D, the right shadow price does exhibit volatility. A fixed tax would not be the optimum.
- Of course the question of “too much” volatility is a question of whether the observed volatility is reflecting institutional frictions and other problems, or reflecting the fundamentals.

The End